Сходится ли ряд:

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots - \frac{(-1)^{n}}{n} \ldots$$

$S = 1 - \frac{1}{2} + \frac{1}{3} - \cdots - \frac{1}{2n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{2n}$

$- 2(\frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2n}) = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{2n}$

$- 2\frac{1}{2}(1 + \frac{1}{2} + \frac{1}{3} + \ldots \frac{1}{n})$

Call $H_n = 1 + \frac{1}{2} + \ldots \frac{1}{n}$ the harmonic series We know that $H_n - \ln n$ converges to the Euler-Mascheroni constant $\gamma = 0.57\ldots$

$S = H_{2n} - H_n = H_{2n} - \ln 2n + H_n + \ln n + \ln 2n - \ln n$

$= (H_{2n} - \ln 2n) - (H_n - \ln n) + ln(\frac{2n}{n}) \to \ln(\frac{2n}{n}) = \ln 2$