

$$\begin{aligned}\sin(x+y) &= \sin x \cos y + \sin y \cos x \\ \sin(x-y) &= \sin x \cos y - \sin y \cos x \\ \cos(x+y) &= \cos x \cos y - \sin y \sin x \\ \cos(x-y) &= \cos x \cos y + \sin y \sin x\end{aligned}$$

$$\begin{aligned}\cos(x+y) + \cos(x-y) &= \cos x \cos y - \sin y \sin x + \cos x \cos y + \sin y \sin x \\ \cos(x+y) + \cos(x-y) &= 2 \cos x \cos y \\ x+y &= a \\ x-y &= b \\ (x+y+x-y) &= a+b = 2x \\ x &= (a+b)/2 \\ 2y &= a-b \\ y &= (a-b)/2 \\ \cos(a) + \cos(b) &= 2 \cos[(a+b)/2] \cos[(a-b)/2]\end{aligned}$$

$$\begin{aligned}\cos(x+y) - \cos(x-y) &= \cos x \cos y - \sin y \sin x - (\cos x \cos y + \sin y \sin x) \\ \cos(x+y) - \cos(x-y) &= -2 \sin y \sin x \\ x+y &= a \\ x-y &= b \\ x &= (a+b)/2 \\ 2y &= a-b \\ y &= (a-b)/2 \\ \cos(a) - \cos(b) &= -2 \sin[(a-b)/2] \sin[(a+b)/2]\end{aligned}$$

$$\begin{aligned}\sin(x+y) + \sin(x-y) &= \sin x \cos y + \sin y \cos x + \sin x \cos y - \sin y \cos x \\ \sin(x+y) + \sin(x-y) &= 2 \sin x \cos y \\ x+y &= a \\ x-y &= b \\ x &= (a+b)/2 \\ 2y &= a-b \\ y &= (a-b)/2 \\ \sin(a) + \sin(b) &= 2 \sin[(a+b)/2] \cos[(a-b)/2]\end{aligned}$$

$$\begin{aligned}\sin(x+y) - \sin(x-y) &= \sin x \cos y + \sin y \cos x - \sin x \cos y - \sin y \cos x \\ \sin(x+y) - \sin(x-y) &= 2 \sin y \cos x \\ x+y &= a \\ x-y &= b \\ x &= (a+b)/2 \\ 2y &= a-b \\ y &= (a-b)/2 \\ \sin(a) - \sin(b) &= 2 \sin[(a-b)/2] \cos[(a+b)/2]\end{aligned}$$

$$\begin{aligned}\cos x \cos y &= (\cos(x+y) + \cos(x-y))/2 \\ \sin x \sin y &= (\sin(x+y) + \sin(x-y))/2 \\ \sin y \cos x &= (\sin(x+y) - \sin(x-y))/2\end{aligned}$$