

$$\sin 4x = (1 + \sqrt{2})(\sin 2x + \cos 2x - 1)$$

$$2 \sin 2x \cos 2x = (1 + \sqrt{2})(\sin 2x + \cos 2x - 1)$$

$$\sin^2(2x) + 2 \sin 2x \cos 2x + \cos^2(2x) = (1 + \sqrt{2})(\sin 2x + \cos 2x - 1) + \sin^2(2x) + \cos^2(2x)$$

$$(\sin 2x + \cos 2x)^2 = (1 + \sqrt{2})(\sin 2x + \cos 2x - 1) + 1$$

$$\sin 2x + \cos 2x = t$$

$$t^2 = (1 + \sqrt{2})(t - 1) + 1$$

$$t^2 - 1 = (1 + \sqrt{2})(t - 1)$$

$$(t - 1)(t + 1) = (1 + \sqrt{2})(t - 1)$$

$$(t - 1) [t + 1 - (1 + \sqrt{2})] = 0$$

$$t = 1$$

$$t = \sqrt{2}$$

$$\sin 2x + \cos 2x = 1$$

$$\sqrt{2} \sin(2x + \pi/4) = 1$$

$$\sin(2x + \pi/4) = 1/\sqrt{2}$$

$$2x + \pi/4 = \pi/4 + 2\pi k$$

$$x = \pi k$$

$$2x + \pi/4 = 3\pi/4 + 2\pi k$$

$$x = \pi/4 + \pi k$$

$$\sin 2x + \cos 2x = \sqrt{2}$$

$$\sqrt{2} \sin(2x + \pi/4) = \sqrt{2}$$

$$\sin(2x + \pi/4) = 1$$

$$2x + \pi/4 = \pi/2 + 2\pi k$$

$$x = \pi/8 + \pi k$$

Ответ  $\{\pi/8 + \pi k\} \cup \{\pi/4 + \pi k\} \cup \{\pi k\}$

