

$$\begin{aligned} \sin 4x &= (1+\sqrt{2})(\sin 2x + \cos 2x - 1) \\ 2\sin 2x \cos 2x &= (1+\sqrt{2})(\sin 2x + \cos 2x - 1) \\ \sin 2x + \cos 2x &= t \\ (\sin 2x + \cos 2x)^2 &= t^2 \\ \sin^2 2x + 2\sin 2x \cos 2x + \cos^2 2x &= t^2 \\ 1 + 2\sin 2x \cos 2x &= t^2 \\ \sin 2x \cos 2x &= (t^2 - 1)/2 \\ t^2 - 1 &= (1+\sqrt{2})(t-1) \end{aligned}$$

$$\begin{aligned} t^2 - 1 &= t - 1 + \sqrt{2}t - \sqrt{2} \\ t^2 - 1 - t + 1 - \sqrt{2}t + \sqrt{2} &= 0 \\ t^2 - t - \sqrt{2}t + \sqrt{2} &= 0 \\ t^2 + t(-1 - \sqrt{2}) + \sqrt{2} &= 0 \\ D &= (-1 - \sqrt{2})^2 - 4\sqrt{2} = (1 + \sqrt{2})^2 - 4\sqrt{2} = 1 + 2\sqrt{2} + 2 - 4\sqrt{2} = 1 - 2\sqrt{2} + 2 = \\ &= (1 - \sqrt{2})^2 \\ x_1 &= \frac{1 + \sqrt{2} + 1 - \sqrt{2}}{2} = 1 \\ x_2 &= \frac{1 + \sqrt{2} - 1 + \sqrt{2}}{2} = \sqrt{2} \end{aligned}$$

$$\begin{aligned} t^2 - 1 &= (1+\sqrt{2})(t-1) \\ (t-1)(t+1) &= (1+\sqrt{2})(t-1) \\ (t-1)(t+1) - (t-1)(1+\sqrt{2}) &= 0 \\ (t-1)(t+1-1-\sqrt{2}) &= 0 \\ (t-1)(t-\sqrt{2}) &= 0 \\ \sin 2x + \cos 2x &= 1 \\ 2\sin x \cos x + 1 - 2\sin^2 x &= 1 \\ 2\sin x \cos x - 2\sin^2 x &= 0 \\ 2\sin x(\cos x - \sin x) &= 0 \\ \sin x &= 0 \\ x &= \pi k \\ \cos x - \sin x &= 0 \\ \cos x &= \sin x \\ 1 &= \sin x / \cos x \\ 1 &= \operatorname{tg} x \\ \operatorname{tg} x &= 1 \\ x &= \pi/4 + \pi k \end{aligned}$$

$$\begin{aligned} f(\sin a + \cos a, \sin a - \cos a) &= 0 \\ \sin a + \cos a &= t \\ \sin a, \cos a &= \frac{t^2 - 1}{2} \end{aligned}$$

- $\sin a + \cos a = t$
- 1) метод вспомогательного аргумента
  - 2) если  $t=0$
- то можно свести к  $\operatorname{tg}$
- 3)  $2\sin(a/2)\cos(a/2) + \cos^2(a/2) - \sin^2(a/2) = t \quad / : \cos^2(a/2)$   
 $2\operatorname{tg}(a/2) + 1 - \operatorname{tg}^2(a/2) = t / \cos^2(a/2)$
- $$1/\cos^2 x = 1 + \operatorname{tg}^2 x$$
- $$2\operatorname{tg}(a/2) + 1 - \operatorname{tg}^2(a/2) = t(1 + \operatorname{tg}^2(a/2))$$
- квадратная ур-ие на  $\operatorname{tg}(a/2)$

$$\begin{aligned} \sin 2x + \cos 2x &= \sqrt{2} \\ \sqrt{2}\sin(2x + \pi/4) &= \sqrt{2} \\ \sin(2x + \pi/4) &= 1 \\ 2x + \pi/4 &= \pi/2 + 2\pi k \\ 2x &= \pi/4 + 2\pi k \\ x &= \pi/8 + \pi k \end{aligned}$$

FINAL ANSWER:  $\pi/8 + \pi k \quad \pi/4 + \pi k \quad \pi k$