

$$\begin{aligned}\sin(x+y) &= \sin x \cos y + \cos x \sin y \\ \sin(x-y) &= \sin x \cos y - \cos x \sin y \\ \cos(x+y) &= \cos x \cos y - \sin x \sin y \\ \cos(x-y) &= \cos x \cos y + \sin x \sin y\end{aligned}$$

$$\begin{aligned}\sin a + \sin b &= \text{????} \\ \sin a - \sin b &= \text{????} \\ \cos a + \cos b &= \text{????} \\ \cos a - \cos b &= \text{????}\end{aligned}$$

$$\sin(x+y) + \sin(x-y) = \sin x \cos y + \cos x \sin y + \sin x \cos y - \cos x \sin y = 2 \sin x \cos y$$

$$\begin{cases} x+y=a \\ x-y=b \end{cases}$$

$$y = a - x$$

$$x - (a - x) = b$$

$$2x - a = b$$

$$2x = a + b$$

$$x = (a+b)/2$$

$$y = a - x$$

$$y = a - (a+b)/2$$

$$y = (2a - a - b)/2$$

$$y = (a-b)/2$$

$$\sin a + \sin b = 2 \sin((a+b)/2) \cos((a-b)/2)$$

$$\sin(x+y) - \sin(x-y) = \sin x \cos y + \sin y \cos x - \sin x \cos y + \sin y \cos x = 2 \sin y \cos x$$

$$a = x + y$$

$$b = x - y$$

$$x = (a+b)/2$$

$$y = (a-b)/2$$

$$\sin a - \sin b = 2 \sin((a-b)/2) \cos((a+b)/2)$$

$$\cos(x+y) + \cos(x-y) = \cos x \cos y - \sin x \sin y + \cos x \cos y + \sin x \sin y = 2 \cos x \cos y$$

$$x + y = a$$

$$x - y = b$$

$$x = (a+b)/2$$

$$y = (a-b)/2$$

$$\cos a \cos b = 2 \cos((a+b)/2) \cos((a-b)/2)$$

$$\cos(x+y) - \cos(x-y) = \cos x \cos y - \sin x \sin y - \cos x \cos y - \sin x \sin y = -2 \sin x \sin y$$

$$x = (a+b)/2$$

$$y = (a-b)/2$$

$$\cos a - \cos b = -2 \sin((a+b)/2) \sin((a-b)/2)$$