

$$\sin x \cdot \cos(x+y) + \sin(x+y) = 3\cos(x+y)$$
$$4\sin x = 5\operatorname{ctg}(x+y)$$

$$\sin x \cdot \cos(x+y) + \sin(x+y) = 3\cos(x+y)$$
$$\sin x \cdot \cos(x+y) + \sin(x+y) - 3\cos(x+y) = 0$$
$$\sin x \cdot \cos(x+y) + \sin x \cdot \cos y + \sin y \cdot \cos x - 3\cos x \cdot \cos y + 3\sin y \cdot \sin x = 0$$
$$\sin x \cdot (\cos x \cdot \cos y - \sin y \cdot \sin x) + \sin x \cdot \cos y + \sin y \cdot \cos x - 3\cos x \cdot \cos y + 3\sin y \cdot \sin x = 0$$
$$\sin 2x/2 - \sin^2 x \cdot \sin y + \sin x \cdot \cos y + \sin y \cdot \cos x - 3\cos x \cdot \cos y + 3\sin y \cdot \sin x = 0$$

$$\sin x \cdot \operatorname{ctg}(x+y) + 1 = 3\operatorname{ctg}(x+y)$$

$$\sin x = a$$
$$\operatorname{ctg}(x+y) = b$$

$$ab + 1 = 3b$$
$$4a = 5b$$

$$ab + 1 = 3b$$
$$a = 5/4b$$

$$5/4b^2 + 1 = 3b$$

$$5b^2 - 12b + 4 = 0$$

$$D/4 = 36 - 20 = 16$$

$$b_1 = (6+4)/5 = 2$$

$$b_2 = (6-4)/5 = 2/5$$

$$a_1 = 10/4 = 5/2$$

$$a_2 = 1/2$$

$\sin x = 5/2$ - НЕВОЗМОЖНО

$$\sin x = 1/2$$

$$x_1 = p/6 + 2pk$$

$$x_2 = 5p/6 + 2pk$$

$$\operatorname{ctg}(x+y) = 2/5$$

$$x+y = \operatorname{arcctg}(2/5) + pn$$

$$y_1 = \operatorname{arcctg}(2/5) + pn - p/6 + 2pk$$

$$y_2 = \operatorname{arcctg}(2/5) + pn - 5p/6 + 2pk$$

Ответ: $(p/6 + 2pk; \operatorname{arcctg}(2/5) + pn - p/6 + 2pk), (5p/6 + 2pk; \operatorname{arcctg}(2/5) + pn - 5p/6 + 2pk)$